Gravitational decoherence of planetary motions

S. Reynaud[1] (*), P. A. Maia Neto[2], A. Lambrecht[1] and M.-T. Jaekel[3]
[1] *Laboratoire Kastler Brossel* (**), *Campus Jussieu case 74, F75252 Paris, France*
[2] *Instituto de Física, UFRJ, Caixa Postal 68528, 21945-970 Rio de Janeiro, Brazil*
[3] *Laboratoire de Physique Théorique* (***), *24 rue Lhomond, F75231 Paris, France*

PACS. 04.30.-w – Gravitational waves: theory.
PACS. 03.65.Yz – Decoherence; open systems; quantum statistical methods.
PACS. 95.10.Ce – Celestial mechanics.

Abstract. – We study the effect of the scattering of gravitational waves on planetary motions, say the motion of the Moon around the Earth. Though this effect has a negligible influence on dissipation, it dominates fluctuations and the associated decoherence mechanism, due to the very high effective temperature of the background of gravitational waves in our galactic environment.

Decoherence plays an important role in the transition between microscopic and macroscopic physics since it kills quantum coherences on a time scale which becomes extremely short for systems with a large degree of classicality [1–4]. The details of this transition depend on the coupling mechanisms between the system under consideration and its environment. A lot of different models have been considered theoretically and decoherence has been experimentally observed in mesoscopic systems, such as a few microwave photons in a high-Q cavity [5], for which the decoherence time is neither too short nor too long.

For large macroscopic masses, say the Moon orbiting around the Earth, decoherence is so efficient that it precludes the observation of quantum coherences. It remains however important from a conceptual point of view to study the dominant decoherence mechanisms and to obtain a reliable estimation of the decoherence time scale. For motions in the solar system, decoherence is often attributed to the scattering of the electromagnetic fluctuations associated with solar radiation or cosmic microwave background. In this letter we show that the decoherence of planetary motions is not dominated by electromagnetic processes but rather by the scattering of stochastic gravitational waves present in our galactic environment.

Gravitational fluctuations have already been proposed as a universal mechanism able to explain the transition from quantum to classical physics. If these fluctuations are characterized by length scales of the order of the Planck length [6, 7], microscopic and macroscopic regions may be delineated by comparing the Planck length $l_{\rm P} = \sqrt{\frac{\hbar G}{c^3}}$ and the Compton length $l_{\rm C} = \frac{\hbar}{mc}$ associated with typical quantum phenomena for a mass m [8–10]. Remarkably, this

(*) E-mail: reynaud@spectro.jussieu.fr
(**) Laboratoire ENS, Université Pierre et Marie Curie, CNRS
(***) Laboratoire CNRS, ENS, Université Paris-Sud

leads to a frontier determined by the Planck mass $m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}}$, i.e. the mass scale built on the Planck constant $\hbar$, the velocity of light c and the Newton constant G with a value 22 μg lying on the borderland between microscopic and macroscopic masses. However, this simple scaling argument is not by itself sufficient to reach any precise conclusion.

In order to compare gravitational and electrodynamical contributions to decoherence, one has to discuss the corresponding fluctuation levels as well as their effects on the system, at the frequencies of interest for the latter. In this letter, we present a quantitative study of decoherence of planetary motions in the stochastic background of gravitational waves in our environment [11, 12]. We give a precise estimate of the associated gravitational decoherence rate and show that it largely dominates the electromagnetic contribution.

The interaction of macroscopic motions with gravitational fluctuations is well known from the theory of gravitational wave emission and gravitational wave detection [13–15]. At the limit of non relativistic velocities, the gravitational perturbation on a planetary system may be represented as a tidal acceleration acting on each mass

$$x_{\mathrm{i}}''(t) = -R_{\mathrm{i0j0}}(t)\, x_{\mathrm{j}}(t) \qquad R_{\mathrm{i0j0}}(t) = -\frac{1}{2} h_{\mathrm{ij}}''(t) \tag{1}$$

The prime denotes a time derivative. The tidal tensor R_{i0j0} is built up from components of the Riemann curvature tensor with the index 0 representing time components and indices i, j representing spatial components. R_{i0j0} has been written as the second order derivative of the metric perturbation h_{ij} evaluated at the center of mass of the planetary system in the transverse traceless (TT) gauge. The interaction is equivalently described by a Lagrangian perturbation coupling the quadrupole momentum of the system to the tidal tensor

$$S'(t) = \frac{1}{4} h_{\mathrm{ij}}''(t)\, Q_{\mathrm{ij}}(t) \qquad Q_{\mathrm{ij}}(t) = m\left(x_{\mathrm{i}}(t)\, x_{\mathrm{j}}(t) - \frac{\delta_{\mathrm{ij}}}{3} x_{\mathrm{k}}(t)\, x_{\mathrm{k}}(t)\right) \tag{2}$$

The quadrupole Q_{ij} is reduced to its traceless part with δ_{ij} a spatial Kronecker symbol. S' is the time derivative of the action integral as it is perturbed along a given trajectory, for example a circular orbit in the plane $x_1 x_2$

$$x_1(t) = \rho \cos(\Omega t + \theta) \qquad x_2(t) = \rho \sin(\Omega t + \theta) \qquad \rho^3 \Omega^2 = GM \tag{3}$$

For a system of two masses m_a and m_b, m denotes the reduced mass $\frac{m_a m_b}{m_a + m_b}$, x_{i} the relative position and ρ the distance between the two masses. The last relation in (3) is the Kepler law which connects the radius ρ, the orbital frequency Ω and the total mass $M = m_a + m_b$.

Our aim is to evaluate decoherence between two neighbouring motions on the same circular orbit. These motions correspond to slightly different values of θ with the difference denoted by $\Delta\theta$. The differential perturbation between the two motions is thus described by a quantity $\Delta S'$ written as in (2) with Q_{ij} replaced by the difference ΔQ_{ij} of the quadrupoles on the two motions. We write it as the product of a distance Δx and a force F

$$\Delta S'(t) = F(t)\, \Delta x \qquad \Delta x = \rho \Delta\theta \qquad F(t) = \frac{1}{2} h_{\mathrm{ij}}''(t)\, m x_{\mathrm{i}}(t) \frac{x_{\mathrm{j}}'(t)}{\rho\Omega} \tag{4}$$

Δx is the distance between the 2 motions, constant on a circular orbit, and F the component of the relative force projected along the mean motion. F may be expressed in terms of the circularly polarized metric perturbation h which fits the circular motion of the planetary system

$$F(t) = \frac{m\rho}{2\sqrt{2}} \left(h'' e^{2i\Omega t} + h^{*\,\prime\prime} e^{-2i\Omega t}\right) \qquad h(t) = \frac{1}{\sqrt{2}} \left(h_{12} + \frac{h_{22} - h_{11}}{2i}\right) \tag{5}$$

The circular polarization h is normalized so that it corresponds to the same noise level as the two linear polarizations h_{12} and $\frac{h_{22}-h_{11}}{2}$ in the case of an unpolarized background. Polarizations are defined with respect to the plane of the orbit and the two motions chosen in the vicinity of $\theta = 0$.

We now come to a Fourier representation of the force F and metric h. Accordingly, force and metric fluctuations are described by noise spectra C_{FF} and C_{hh}

$$F[\omega] = \int \mathrm{d}t \; F(t) \; e^{i\omega t} \qquad C_{FF}[\omega] = \int \mathrm{d}t \; \langle F(t) \cdot F(0) \rangle \; e^{i\omega t} \tag{6}$$

The dot specifies a symmetrical ordering when quantum fluctuations are considered [16, 17]. The force (5) is driven by gravitational waves through a frequency transposition due to the evolution of the quadrupole momentum at frequencies $\pm 2\Omega$

$$F[\omega] = -m\rho \frac{(\omega + 2\Omega)^2 h[\omega + 2\Omega] + (\omega - 2\Omega)^2 h^* [\omega - 2\Omega]}{2\sqrt{2}} \tag{7}$$

We are mainly interested in the long term cumulative effect of fluctuations, that is in the momentum perturbation p_t integrated over an interaction time t longer than the correlation time. The variance of p_t is determined by the noise spectrum $C_{FF}[0]$ evaluated at zero frequency and expressed in terms of a momentum diffusion coefficient D

$$p_t = \int_0^t \mathrm{d}s \; F(s) \qquad \left\langle p_t^2 \right\rangle = C_{FF}[0] \; t = 2Dt \tag{8}$$

The diffusion coefficient D_{gr} due to gravitational waves is finally obtained as

$$2D_{\mathrm{gr}} = C_{FF}[0] = 4m^2 a^2 C_{hh}[2\Omega] \qquad a = \rho\Omega^2 \tag{9}$$

a is the acceleration on the circular orbit and $C_{hh}[2\Omega]$ is the gravitational wave spectrum at frequency 2Ω of the circular polarization h coupled to the system. Note that a gravitational background of galactic origin is certainly not isotropic whereas an extragalactic background may probably be considered as isotropic. For simplicity, forthcoming discussions are phrased in terms of an unpolarized and isotropic background but the general case is recovered by coming back to equation (9).

The frequency of interest for the planetary motion of the Moon is $\frac{2\Omega}{2\pi} \simeq 0.8 \times 10^{-6}$Hz. Information on stochastic gravitational waves around this frequency may be deduced from studies devoted to the detectability of gravitational background by interferometers [18, 19]. We express the spectrum C_{hh} as a gravitational wave energy $k_{\mathrm{B}} T_{\mathrm{gr}}$ or, equivalently, as a number n_{gr} of gravitons per mode

$$C_{hh}[\omega] = \frac{16G}{5c^5} k_{\mathrm{B}} T_{\mathrm{gr}}[\omega] = \frac{16G}{5c^5} \left(\frac{1}{2} + n_{\mathrm{gr}}[\omega] \right) \hbar\omega \tag{10}$$

k_{B} is the Boltzmann constant and T_{gr} is an effective temperature of the gravitational background. Figure 1 of [18] and equation (3.1) of [19] lead respectively to $C_{hh}[2\Omega] \simeq 10^{-34.5}\mathrm{Hz}^{-1}$ and $C_{hh}[2\Omega] \simeq 10^{-33}\mathrm{Hz}^{-1}$ for $\frac{2\Omega}{2\pi} \simeq 10^{-6}$Hz. Taking as a conservative estimate $C_{hh} \simeq 10^{-34}\mathrm{Hz}^{-1}$, we obtain an effective temperature $T_{\mathrm{gr}} \approx 10^{41}$K or, equivalently, a graviton number per mode $n_{\mathrm{gr}} \approx 2 \times 10^{57}$. These numbers correspond to the high temperature limit $k_{\mathrm{B}} T_{\mathrm{gr}} \gg \hbar\omega$. Hence the gravitational noise is much larger than vacuum fluctuations which correspond to the term $\frac{1}{2}$ in (10) and lead to ultimate fluctuations of geodesic distances of the order of Planck length [17].

The estimations of C_{hh} used here correspond to the confusion background of gravitational waves emitted by binary systems in our galaxy. They rely on the laws of physics and astrophysics as they are known in our local celestial environment. There also exist more speculative predictions for gravitational backgrounds emitted by cosmic processes [18–21]. Depending on the parameters used in the models, these cosmic backgrounds may surpass the binary confusion background in the μHz frequency range. Hence the latter can be considered as a minimum noise level in our gravitational environment.

The diffusion coefficient may be written under the form of an Einstein fluctuation-dissipation relation, i.e. as the product of the effective temperature by a damping factor

$$D_{\mathrm{gr}} = m\Gamma_{\mathrm{gr}} k_{\mathrm{B}} T_{\mathrm{gr}} \qquad \Gamma_{\mathrm{gr}} = \frac{32Gma^2}{5c^5} \tag{11}$$

The damping rate Γ_{gr} is the inverse of the damping time associated with the emission of gravitational waves by the planetary system [13–15]. It does not depend on temperature and is so small, $\Gamma_{\mathrm{gr}} \approx 10^{-34}\mathrm{s}^{-1}$ for Moon, that it does not affect the classical motion on the orbit. Gravitational damping however has a noticeable effect in strongly bound binary systems such as millisecond pulsars [22].

At this point it is worth comparing the effects of gravitational and electromagnetic scattering. Modelling the moving mass as a sphere which perfectly scatters thermal photons at temperature T_{em}, the damping rate Γ_{em} associated with radiation pressure of these electromagnetic fluctuations is evaluated as [23]

$$\Gamma_{\mathrm{em}} = \frac{4\pi^3 \hbar r^2}{45m} \left(\frac{k_{\mathrm{B}} T_{\mathrm{em}}}{\hbar c} \right)^4 \tag{12}$$

The radius r has been supposed to be large compared to the wavelength of the photons. For the Moon, the temperature $T_{\mathrm{em}} = 2.7$K of the cosmic microwave background is already sufficient to produce a damping rate $\Gamma_{\mathrm{em}} \approx 2 \times 10^{-32}\mathrm{s}^{-1}$ which is 200 times larger than Γ_{gr}. Since Γ_{em} varies as T_{em}^4 in agreement with Stefan-Boltzmann law, the damping due to solar radiation is more than 10^{10} times larger than Γ_{gr}. In fact, the damping of the Moon, as it is revealed by the secular variation of lunar rotation [24], is mainly due to the interaction between Earth and Moon tides and it corresponds to a damping rate more than 10^{16} times larger than Γ_{gr}. The role of gravitational scattering on damping of the Moon may thus be ignored but this is no longer the case for decoherence, as is shown in next paragraphs.

In order to evaluate decoherence rates, we consider the effect of gravitational perturbation on the action difference ΔS_t after an interaction time t

$$\Delta S_t = \int_0^t \mathrm{d}s F(s)\, \Delta x = p_t \Delta x \qquad \left\langle \Delta S_t^2 \right\rangle = \left\langle p_t^2 \right\rangle \Delta x^2 \tag{13}$$

The decoherence between the two neighbouring trajectories is measured by the mean value $\left\langle \exp \frac{i\Delta S_t}{\hbar} \right\rangle$ of the exponentiated dephasing. Since ΔS_t is linearly driven by gravitational waves, it can be treated as a gaussian classical stochastic variable which leads to

$$\left\langle \exp \frac{i\Delta S_t}{\hbar} \right\rangle = \exp\left(-\frac{\left\langle \Delta S_t^2 \right\rangle}{2\hbar^2} \right) = \exp\left(-\Lambda_{\mathrm{gr}} \Delta x^2 t \right)$$

$$\Lambda_{\mathrm{gr}} = \frac{D_{\mathrm{gr}}}{\hbar^2} = \frac{32Gm^2a^2}{5c^5\hbar^2} k_{\mathrm{B}} T_{\mathrm{gr}} \tag{14}$$

Decoherence has been evaluated in the long term limit, where the force fluctuations may be approximated as a white noise characterized by a momentum diffusion coefficient D_{gr}. This expression can also be derived by evaluating the Feynman-Vernon influence functionals [25], often used in the study of decoherence [26], at the limit of high temperature. Note that the decoherence rate Λ_{gr} is proportional to the square of the acceleration and, hence, would vanish if evaluated for an inertial motion. This has a simple interpretation through the Einstein formula (11). The damping rate Γ_{gr} associated with the emission of gravitational waves indeed vanishes for inertial motion while the other factors entering the expression of D_{gr} do not depend on the specific motion.

Using the Einstein fluctuation-dissipation relation (11) and the expression (14) of decoherence rates, the ratio between gravitational and electromagnetic contributions to decoherence can be rewritten

$$\frac{\Lambda_{\mathrm{gr}}}{\Lambda_{\mathrm{em}}} = \frac{D_{\mathrm{gr}}}{D_{\mathrm{em}}} = \frac{\Gamma_{\mathrm{gr}}}{\Gamma_{\mathrm{em}}}\frac{T_{\mathrm{gr}}}{T_{\mathrm{em}}} \tag{15}$$

For the motion of the Moon, the gravitational decoherence rate Λ_{gr} is found to be 10^{38} larger than the rate Λ_{em} corresponding to scattering of the cosmic photon background. It is still enormously larger than the effect corresponding to the scattering of solar photons. The same conclusion is reached for the comparison with the effect of tides. The latter effect determines the damping of the main motion of Moon but its contribution to decoherence is overshadowed by gravitational scattering owe to the very high value of the effective gravitational temperature. As a consequence of this high temperature, the decoherence time is exceedingly short even for ultrasmall distances Δx. To fix ideas, the gravitational decoherence rate $\Lambda_{\mathrm{gr}} \approx 10^{75}\mathrm{s}^{-1}\mathrm{m}^{-2}$ obtained for the motion of the Moon corresponds to a decoherence time in the 10μs range for a distance Δx between two trajectories as small as Planck length.

The large value of gravitational temperature implies that decoherence should still be dominated by gravitational scattering for smaller size planetary systems. This can be discussed by writing the ratio between gravitational and electromagnetic decoherence rates as a product of dimensionless factors

$$\frac{\Lambda_{\mathrm{gr}}}{\Lambda_{\mathrm{em}}} = \frac{72}{\pi^3}\frac{m^2}{m_{\mathrm{P}}^2}\frac{\rho^2}{r^2}\left(\frac{\hbar\Omega}{k_{\mathrm{B}}T_{\mathrm{em}}}\right)^4\frac{T_{\mathrm{gr}}}{T_{\mathrm{em}}} \tag{16}$$

The factor $\frac{m^2}{m_{\mathrm{P}}^2}$ is clearly reminiscent of the simple scaling arguments presented in the Introduction, illustrating the role of Planck mass as a reference on the borderland between microscopic and macroscopic masses. However the whole formula shows that these scaling arguments are not sufficient for obtaining reliable quantitative predictions. The factor $\frac{\rho^2}{r^2}$ is a geometrical factor depending on the radius ρ of the orbit and the radius r of the orbiter. Then the ratio (16) also depends on the inverse fourth power of the photon number $\frac{k_{\mathrm{B}}T_{\mathrm{em}}}{\hbar\Omega}$ per mode at the orbital frequency Ω and on the already discussed ratio between temperatures of graviton background and photon background. Clearly the last three dimensionless factors appearing in (16) have no relation with the scaling arguments of the Introduction but have to be known for a quantitative comparison of gravitational and electromagnetic contributions. If we consider as an example a man made gravitationally bound planetary system consisting of two spheres having an ordinary metallic density, we obtain comparable decoherence rates for gravitational scattering and electromagnetic scattering of cosmic microwave background in the case of a compact geometry with masses of the order of 10^3kg.

We have shown in this letter that the scattering of gravitational waves in our celestial environment is a dominant cause of decoherence for planetary motions such as the motion

of the Moon around the Earth. Due to the very large effective temperature of the gravitational wave background, this mechanism leads to a decoherence by far more efficient than the other fluctuation mechanisms though its contribution to damping of the mean motion can be neglected. As far as the theory of measurement is concerned, this implies that the ultimate fluctuations of the motion of Moon are determined by the same classical gravitation theory which also explains its mean motion. Precisely these fluctuations are determined by the classical gravitational waves present in our local celestial environment.

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P.A.M.N. wishes to thank CAPES, CNPq, FAPERJ, PRONEX, COFECUB, ENS and MENRT for their financial support which made possible his stays in Paris during which this work was performed.

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